SyDe312 Numerical Methods - Test 3

18 March 2005, 12:30-1:20

Read the questions CAREFULLY. You can easily make them much more difficult and much longer than intended!

1. [7 marks]

   (a) What is a linear least squares problem why is it called *linear*?

   (b) Describe briefly three different methods that could be used to solve a linear least squares problem and explain the merits of each.

   (c) Find a fitting function of the form $f(x) = a/x + b/x^2 + c$ for the following (x, y) data:

x	1.0	1.5	2.0	3.2	4.1	6.2
y	1.9403	1.3020	0.8022	0.4217	0.3770	0.2275

   (d) What is the residual of your answer to part (c)?

2. [7 marks]

   (a) For a set of (x, y) data describe the following and explain how they are different or the same: (i) a cubic spline, (ii) a cubic Lagrange interpolating polynomial, (iii) a cubic Newton interpolating polynomial and (iv) a cubic least squares fitting polynomial? In your answer be sure to consider all possibilities for the number of data points.

   (b) Derive BY HAND CALCULATION the equation of ONE segment of the *natural* cubic spline $y = f(x)$ applicable to the value $x = 2.6$ interpolating the data in problem 1(c). You can (should) use matlab to solve any linear system that arises!

   (c) Use your spline equation from part (b) to evaluate $f(2.6)$.

   (d) Use the matlab `spline` function or `interp1` to generate a cubic spline $g(x)$ for the same data, using zero endpoint slope conditions and evaluate $g(2.6)$.

   (e) Do different endpoint constraints affect the value of the spline function at points that don't lie in an end segment? If so why?

3. [1 mark] Which of the various possibilities in problems 1 and 2 would be most appropriate for analysing the data in problem 1c) and why?

FORMULAS ON REVERSE ...... TURN OVER

$$y = Ay_j + By_{j+1} + Cy''_j + Dy''_{j+1}$$

$$\begin{bmatrix} h_1 & 2(h_1+h_2) & h_2 & & & & \\ & h_2 & 2(h_2+h_3) & h_3 & & & \\ & & & \ddots & & & \\ & & & & h_{n-2} & 2(h_{n-2}+h_{n-1}) & h_{n-1} \end{bmatrix} \begin{bmatrix} y''_1 \\ y''_2 \\ y''_3 \\ \vdots \\ y''_{n-1} \\ y''_n \end{bmatrix}$$

$$= 6 \begin{bmatrix} f[x_2,x_3] - f[x_1,x_2] \\ f[x_3,x_4] - f[x_2,x_3] \\ \vdots \\ f[x_{n-2},x_{n-1}] - f[x_{n-3},x_{n-2}] \end{bmatrix}$$

$$\begin{aligned} h_j &= x_{j+1} - x_j, j = 1, 2, \cdots, n-1 \\ A &= (x_{j+1} - x)/h_j \\ B &= 1 - A \\ C &= (1/6)(A^3 - A)h_j^2 \\ D &= (1/6)(B^3 - B)h_j^2 \end{aligned}$$